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PSEUDOSPHERE ARRANGEMENTS WITH SIMPLE COMPLEMENTS

LEWIS PAKULA

ABSTRACT. A necessary and sufficient condition for the complements of a general kind of topological sphere arrangement (and those of all its subarrangements) to have homologically trivial components is that all intersection degeneracies be nonnegative in a certain sense. Transverse intersections are not assumed. This extends the domain of application of formulas which count these components in terms of degeneracies. In dimension 2 we show that the union of pseudocircles in an arrangement with possibly nontransverse intersections is the same as the union of an arrangement with only transverse intersections.

1. Introduction and main result. Formulas which count the regions into which $\mathbf{R}^n$ is subdivided by a collection of Euclidean hyperplanes in general position have long been known, see, e.g., [**1**], but neat formulas when the hyperplanes are not in general position are of surprisingly recent vintage [**5, 6**]. It turns out that such formulas apply more generally to certain arrangements of topological spheres. The formulas follow from additivity of the Euler characteristic once it is established that complement components of the arrangement and its subarrangements have the homology of a point. We will characterize such arrangements.

We consider indexed families of subsets of an n-sphere which intersect topologically like Euclidean spheres in a certain sense. Denote the index set $\{1, \ldots, k\}$ by $[k]$.

Suppose A_i is a closed subset of a topological n-sphere $\mathcal{S}^n$ for each $i \in [k]$, and let $I \subseteq [k]$. We denote $\cap_{i\in I} A_i$ by A_I for $I \neq \varnothing$ and set $A_\varnothing := \mathcal{S}^n$. If each A_I (and, in particular, each A_i, $i \in [k]$) is either a single point or is homeomorphic to a sphere of some dimension, we call the indexed family $\mathcal{A} = \{A_i : i \in [k]\}$ a *pseudosphere arrangement* in $\mathcal{S}^n$. (The empty set is a sphere of dimension -1.)

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