ROCKY MOUNTAIN
JOURNAL OF MATHEMATICS
Volume 33, Number 4, Winter 2003

POSITIVE SOLUTIONS OF NONLINEAR FOCAL BOUNDARY VALUE PROBLEMS

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ABSTRACT. This paper investigates the nth order ordinary differential equation: $-x^{(n)} = f(t, x, x', x'', \dots, x^{(n-1)})$ with focal boundary value conditions. We give some estimation results to deal with the term $x^{(n-1)}$ which appeared in f. By using the fixed point index theory, we obtain the existence of positive and multiple positive solutions.

1. Introduction. In this paper we consider the existence of positive solutions and multiple positive solutions of

$$-x^{(n)} = f(t, x, x', x'', \dots, x^{(n-1)}), \quad 0 < t < 1, \tag{1.1}$$

with focal boundary value conditions

$$\begin{aligned} x^{(r_i-1)}(0) &= 0, \quad 1 \le i \le k; \\ x^{(s_j-1)}(1) &= 0, \quad 1 \le j \le n-k, \end{aligned} \tag{1.2}$$

where $\{r_1, \dots, r_k\}$ and $\{s_1, \dots, s_{n-k}\}$ form a disjoint partition of $\{1, 2, \dots, n\}$ such that $r_1 < \dots < r_k$ and $s_1 < \dots < s_{n-k}$. For each $0 \le i \le n-1$, define

$$\sigma_i = \operatorname{card}\{j \mid s_j > i\} + 1. \tag{1.3}$$

We assume throughout that

(i) $f \in C[I \times K, R_+]$ where $I = [0,1]$, $R_+ = [0, +\infty)$ and $K = (-1)^{\sigma_0} R_+ \times (-1)^{\sigma_1} R_+ \times (-1)^{\sigma_2} R_+ \times \dots \times (-1)^{\sigma_{n-2}} R_+ \times R^1$.

(ii) $\{r_{k-1}, r_k\} \ne \{n-1, n\}$, $\{s_{n-k-1}, s_{n-k}\} \ne \{n-1, n\}$.

1991 AMS *Mathematics Subject Classification.* 34B15.
Key words and phrases. Focal boundary value problems, positive solutions, Green's function, fixed point index.
Received by the editors on August 15, 2000, and in revised form on April 24, 2001.