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AN OSCILLATION THEOREM FOR DISCRETE EIGENVALUE PROBLEMS

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ABSTRACT. In this paper we consider problems that consist of symplectic difference systems depending on an eigenvalue parameter, together with self-adjoint boundary conditions. Such symplectic difference systems contain as important cases linear Hamiltonian difference systems and also Sturm-Liouville difference equations of second and of higher order. The main result of this paper is an oscillation theorem that relates the number of eigenvalues to the number of generalized zeros of solutions.

1. Introduction. Consider the symplectic difference system

$$z_{k+1} = \mathcal{S}_k z_k, \quad k \in \mathbf{Z}, \tag{S}$$

where the $2n \times 2n$ matrices $\mathcal{S}_k$ are symplectic, i.e.,

$$\mathcal{S}_k^T \mathcal{J} \mathcal{S}_k = \mathcal{J} \quad \text{with} \quad \mathcal{J} = \begin{pmatrix} 0 & I \\ -I & 0 \end{pmatrix}.$$

Symplectic difference systems (S) cover a large variety of difference equations and systems, among them also linear Hamiltonian difference systems

$$\Delta x_k = A_k x_{k+1} + B_k u_k, \quad \Delta u_k = C_k x_{k+1} - A_k^T u_k,$$

where the $n \times n$ matrices B_k and C_k are symmetric and $I - A_k$ is nonsingular, as discussed, e.g., in the monograph by Ahlbrandt and Peterson [**2**]. This means, in turn, that systems (S) also cover higher order Sturm-Liouville difference equations

$$\sum_{\mu=0}^{n} (-\Delta)^\mu \{r_\mu(k) \Delta^\mu y_{k+1-\mu}\} = 0 \quad \text{with} \quad r_n(k) \neq 0,$$

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