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COMMON FIXED POINT THEOREMS FOR WEAKLY COMPATIBLE MAPPINGS

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ABSTRACT. This work is a continuation of [**18,19,26–28**]. The concept of weak compatibility between a set-valued mapping and a single-valued mapping of Jungck and Rhoades [**19**] is used as a tool for proving some common fixed point theorems on metric spaces. Generalizations of known results, especially theorems by Fisher [**7**], are thereby obtained. As an application of this generalization, one example is given.

1. Introduction. In 1922, the Polish mathematician, Banach, proved a theorem which ensures, under appropriate conditions, the existence and uniqueness of a fixed point. His result is called Banach's fixed point theorem or the Banach contraction principle. This theorem provides a technique for solving a variety of applied problems in mathematical science and engineering. Many authors have extended, generalized and improved Banach's fixed point theorem in different ways. In [**11**], Jungck introduced more generalized commuting mappings, called *compatible mappings*, which are more general than commuting and weakly commuting mappings (Definition 1.4). This concept has been useful for obtaining more comprehensive fixed point theorems (see, e.g., [**1, 2, 4, 5, 9–18, 20–25, 29, 32, 34, 35**]).

Recently, Jungck and Rhoades [**18, 19**] defined the concepts of δ-compatible and weakly compatible mappings which extend the concept of compatible mappings in the single-valued setting to set-valued mappings. Several authors used these concepts to prove some common fixed point theorems (see, e.g., [**18, 19, 26–28**]).

Throughout this paper, let (X, d) be a complete metric space unless mentioned otherwise and $B(X)$ is the set of all nonempty bounded subsets of X. As in [**6, 8**], let $\delta(A, B)$ and $D(A, B)$ be the functions

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