ROCKY MOUNTAIN
JOURNAL OF MATHEMATICS
Volume 20, Number 1, Winter 1990

TRUNCATED ERGODIC THEOREMS FOR NON-SINGULAR AUTOMORPHISMS

CESAR E. SILVA

1. Introduction. In this paper we study ergodic theorems for invertible measurable non-singular transformations. We use a skew product construction introduced by Maharam in [**6**] to show the convergence of some new ergodic ratios and the existence of subsequences of integers for which a known ergodic ratio converges. We then look at limits of truncated ergodic ratios, and study these limits as the constants that bound the Radon-Nikodym derivatives increase to infinity. The study of these limits is motivated by an attempt to obtain the Hurewicz-Halmos-Oxtoby ergodic theorem for non-singular transformations (Theorem 1.2) from the Hopf ergodic theorem for measure preserving transformations (Theorem 1.1). We obtain some partial results in this direction. For example, we show (Theorem 3.2) that, for type III_1 automorphisms, a truncated limit of Maharam (Theorem 3.1) is in fact equal to the Hurewicz-Halmos-Oxtoby limit. (Theorem 3.2 has also been obtained independently by D. Maharam (unpublished).) The last section studies other related truncated limits.

Henceforth (X, B, μ) will denote a σ-finite measure space; sometimes we shall simply write X or (X, μ) to denote this space. A *non-singular automorphism* of (X, B, μ) is an invertible transformation T such that A is measurable if and only if TA is measurable and A is null if and only if TA is null. For any integer $n, \mu T^n$ is a measure and there exist Radon-Nikodym derivatives $\omega_n(x) = d\mu T^n/d\mu(x)$. We usually write ω_1 as ω. One can show that the following relation holds a.e.:

$$\omega_{i+j}(x) = \omega_j(x)\ \omega_i(T^j x). \tag{1.1}$$

A non-null set W is said to be *wandering* if $T^{-n}W \cap W = \varnothing$ for $n > 0$. An automorphism T is *conservative* (or *incompressible*) if it

1980 *Subject Classification*: 28D.
Supported in part by a Williams College Faculty Research Grant.
Received by the editors on August 23, 1985 and in revised form on September 2, 1987.