ROCKY MOUNTAIN
JOURNAL OF MATHEMATICS
Volume 20, Number 1, Winter 1990

CONTROLLING CONJUGACY CLASSES IN EMBEDDINGS OF LOCALLY FINITE GROUPS

SUSAN E. SCHUUR

Several recent papers in the theory of locally finite groups discuss the concept of π-homogeneity [**1**, **2**, **3**]. A locally finite group G is *π-homogeneous* for the set of primes π if, for every isomorphism $\mu : H \to K$ between finite π-subgroups of G, there is an $x \in G$ with $h\mu = h^x$ for all $h \in H$. The group G is a *π-ULF group* if it is locally finite, π-homogeneous and contains a copy of every finite group. One of the results of [**3**] is that any locally finite group G can be embedded in a π-ULF group of cardinality $\max\{\aleph_0, |G|\}$ in which, for each $p \notin \pi$, there are exactly two conjugacy classes of elements of order p. The purpose of this paper is to extend this result as follows:

THEOREM. *Let G be a locally finite group and π a non-empty set of primes. Let $K = \{k_p | p \in \pi_1'\}$ be a set of positive integers, where $\pi_1' \subseteq \pi'$. Then there is a π-ULF group $\overline{G}$ satisfying*

(i) $G \subseteq \overline{G}$ *and* $|\overline{G}| = \max\{\aleph_0, |G|\}$;

(ii) *if $p \in \pi'$, $n \geq 1$, and $\nu(p^n, \overline{G})$ is the set of $\overline{G}$-conjugacy classes of elements of order p^n, then*

$$|\nu(p^n, \overline{G})| = \begin{cases} k_p + 1, & \text{if } p \in \pi_1' \text{ and } n > k_p, \\ n + 1, & \text{otherwise .} \end{cases}$$

This theorem was suggested by one of the constructions in [**3**] (cf. Theorem 3).

We need the following notation. In any group G, $\sim_G$ denotes G-conjugacy of elements and subgroups. If $H \subseteq G$, $(G : H)$ denotes the set of right cosets of H in G and $\Sigma = \Sigma(G : H)$ the full symmetric group on $(G : H)$. The representation $\varphi = \varphi(G : H)$ of G into $\Sigma(G : H)$ is defined by

$$Hx(g\varphi) = Hxg \quad \text{for } x, g \in G;$$

Received by the editors on December 29, 1986.