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CURVATURE AND PROPER HOLOMORPHIC MAPPINGS BETWEEN BOUNDED DOMAINS IN $\mathbf{C}^n$

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ABSTRACT. In this paper we discuss some connections between proper holomorphic mappings between domains in $\mathbf{C}^n$ and the boundary behaviors of certain canonical invariant metrics (Cheng-Yau-Einstein Kähler metric, Bergman metric, intrinsic measures, etc.). Some compactness theorems have been proved (Theorem 4, Theorem 5). This generalizes an earlier result proved by the second author.

Introduction. A continuous mapping $f : X_1 \to X_2$ between two topological spaces is called proper if $f^{-1}(K) \subset X_1$ is compact whenever $K \subset X_2$ is compact. Proper holomorphic mappings between analytic spaces stand out for their beauty and simplicity. For instance, if $g : D_1 \to D_2$ is a proper holomorphic mapping between two bounded domains in C^n, a theorem of Remmert says that (D_1, g, D_2) is a finite branching cover. The branching locus in D_1 is described by $\{z \in D_1 | \det(dg(z)) = 0\}$. For the past ten years, there was a great amount of activity in characterizing the proper holomorphic mappings between pseudoconvex domains. It has been known for a long time that there are numerous proper holomorphic maps between unit disks in $\mathbf{C}^1$. The simplest example is $g : \Delta = \{z \in \mathbf{C}^1 | |z| < 1\} \to \Delta, g(z) = z^n$, where n is any positive integer. Nevertheless, such a phenomenon is no longer true in higher dimensional cases. H. Alexander was able to verify the following interesting fact.

THEOREM 1. [**1**]. *Let* $B_n = \{(z_1, z_2, \ldots, z_n) | \sum_{i=1}^n |z_i|^2 < 1\}$ *be the unit ball in* $\mathbf{C}^n, n \geq 2$. *Suppose* $f : B_n \to B_n$ *is a proper holomorphic mapping; then* f *must be a biholomorphism.*

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