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ON THE SPACE OF VECTOR-VALUED FUNCTIONS OF BOUNDED VARIATION

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ABSTRACT. If the space of all real-valued functions of bounded variation on a real closed interval is endowed with the topology of simple convergence, then every bounded subset which is bounded for the values of total variation is relatively sequentially compact by Helly's selection principle. In this paper, by treating vector-valued functions on a linearly ordered set, we consider an extension of this classical result.

1. Introduction. If the space $\mathrm{BV}[a,b]$ of all real-valued functions of bounded variation on the real closed interval $[a,b]$ is endowed with the topology of simple convergence, then, by Helly [**1**], every subset of $\mathrm{BV}[a,b]$, of the form $\{f|\max|f(x)| \leqq C, V_b^a[f] \leqq K\}$, is sequentially compact, where the positive numbers C, K are constants and $V_b^a[f]$ denotes the total variation of f.

In this paper, to give an interesting extension of this classical result, we consider a space of vector-valued functions on a linearly ordered interval of bounded variation, which take values in a locally convex space E. Then we examine conditions on E under which analogs of Helly's result are valid.

Let $E(\tau)$ be a sequentially complete Hausdorff locally convex space over the real or complex field. E' denotes the dual of $E(\tau)$. We write $\Gamma = \{p_\lambda | \lambda \in \Lambda\}$ for a system of saturated semi-norms on E generating the topology τ. A linearly ordered interval with a maximum and a minimum element is denoted by $[a,b]$ and its cardinal number is denoted by $\overline{\overline{[a,b]}}$. For simplicity we write $F(\tau_p)$ for the product space $\prod_{\alpha\in[a,b]} E_\alpha(\tau_\alpha)$, where $E_\alpha(\tau_\alpha) = E(\tau)$ for all $\alpha \in [a,b]$ and write F' for the direct sum $\oplus_{\alpha\in[a,b]} E'_\alpha$. The reader is referred to [**4**] for terminology used in this article.

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