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THE INTEGERS OF A CYCLIC QUARTIC FIELD

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ABSTRACT. A simple explicit integral basis is given for a cyclic quartic extension of the rationals.

In [**3**] the authors show that a cyclic quartic extension K of the rational number field Q can be expressed uniquely in the form

$$K = Q\left(\sqrt{A(D+B\sqrt{D})}\right), \tag{1}$$

where A, B, C, D are integers such that

$$A \text{ is squarefree and odd,} \tag{2}$$

$$D = B^2 + C^2 \text{ is squarefree}, \qquad B > 0, C > 0, \tag{3}$$

$$GCD(A, D) = 1. \tag{4}$$

This representation of K is simpler than those given in [**2**] and [**4**]. The field K is totally real if $A > 0$ and totally imaginary if $A < 0$. It is also shown in [**3**] that the discriminant $d(K)$ of K is given by

$$d(K) = \begin{cases} 2^8A^2D^3, & \text{if } D \equiv 0 \pmod 2, \\ 2^6A^2D^3, & \text{if } D \equiv 1 \pmod 2,\ B \equiv 1 \pmod 2, \\ 2^4A^2D^3, & \text{if } D \equiv 1 \pmod 2,\ B \equiv 0 \pmod 2, \\ & \quad A+B \equiv 3 \pmod 4, \\ A^2D^3, & \text{if } D \equiv 1 \pmod 2,\ B \equiv 0 \pmod 2, \\ & \quad A+B \equiv 1 \pmod 4. \end{cases} \tag{5}$$

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