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THE SQUARE CLASS INVARIANT

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A. Solow introduced the square class invariant in [**6**] and showed in [**6, 7, 8**] that the square class invariant or the square class invariant plus the determinant classify quadratic forms over some particular fields. It was hoped that this new invariant might be helpful in the classification problem. However, in this paper we will determine all non-pythagorean fields over which the square class invariant classifies quadratic forms and will see that the answer is tied very closely to value sets of forms. In what appears below, the notation follows that in [**4**].

DEFINITION. Let q be a quadratic form over a field F. The square class invariant for q is a function $m_q : \dot{F}/\dot{F}^2 \to \mathbf{Z}$ given by $m_q(a\dot{F}^2) = n$ where $q \cong n\langle a\rangle + p$ and $a \notin D(p)$.

It is clear that m_q is related closely to the value set of q. Indeed, $D(q) = \{a \in \dot{F} | m_q(a) \geq 1\}$ where we write $m_q(a)$ for $m_q(a\dot{F}^2)$. We will show that when the square class invariant classifies forms for non-pythagorean fields, the field's anisotropic forms are determined uniquely by their value sets. Such fields are called C-fields and were introduced in [**2**]. Although there exist C-fields when the level, $s(F)$, is greater than 2, the square class invariant fails to classify forms over any non-pythagorean field when the level is greater than 2. We suspect the complete answer is that the square class invariant classifies quadratic forms over a non-pythagorean field if and only if F is a C-field and $s(F) \leq 2$. Below we will verify this except for one direction in the case when $s(F) = 2$ and both $u(F)$ and the index of $D\langle 1, 1\rangle$ in $\dot{F}$ are infinite.

PROPOSITION 1. *Let F be a field with $s(F) = 1$. Then the square class invariant classifies forms over F if and only if F is a C-field.*

PROOF. Assume the square class invariant classifies forms over F.

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