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THE AUTOMORPHISM GROUPS OF THE HYPERELLIPTIC SURFACES

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1. Introduction. In this paper we will compute the automorphism groups of the so-called hyperelliptic surfaces. These compact complex surfaces are characterized by having invariants $p_g = 0$, $q = 1$, and $12K = 0$. References for the elementary properties of these surfaces may be found in [**2**] (where they are called "bielliptic surfaces") or in [**1**]. They may all be constructed as the quotient $X = (E \times F)/G$, where E and F are elliptic curves, and G is a finite group of translations of E acting also on F *not* only as a group of translations; the action on $E \times F$ is the diagonal action.

There are seven non-isomorphic groups G which can act on $E \times F$ as above, two of which act on any $E \times F$, the other five requiring F to be a specific elliptic curve. In the following table the reader will find a list of the seven groups G, together with the elliptic curves E and F, and the action of G on $E \times F$.

Write $E = \mathbf{C}/(\mathbf{Z} + \mathbf{Z}\tau_1)$ and $F = \mathbf{C}/(\mathbf{Z} + \mathbf{Z}\tau_2)$. Throughout this article we will use the notation $i = \sqrt{-1}, \omega = e^{2\pi i/3}$, and $\zeta = e^{\pi i/3}$; note that $\omega = \zeta^2$.

In the last three cases it is technically more convenient to consider $X = (E \times F)/G$ as the quotient of $Y = (E \times F)/\langle \psi \rangle$ by a cyclic group of order $r(= 2, 3, 4, \text{ or } 6)$, generated by the automorphism $\bar{\phi}$ induced by ϕ. Since ψ is a translation of $E \times F, Y$ is also a complex torus of dimension two. For uniformity of notation we will define $Y = E \times F$ and $\psi =$ identity in the first four cases, so that in each case $X = Y/\langle \bar{\phi} \rangle$. Note that r is the order of the canonical class K_X in $\operatorname{Pic}(X)$ and Y is the etale cyclic cover of X defined by $K_X : Y = \operatorname{Spec}(\oplus_{i=0}^{r-1} \varphi_X(iK_X))$, with the multiplication in φ_Y defined by a chosen isomorphism $\theta : \varphi_X \to \varphi_X(rK_X)$. The formation of Y

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