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ZERO-ONE SET FUNCTIONS AND ABSOLUTE CONTINUITY

WILLIAM D.L. APPLING

ABSTRACT. Suppose that U is a set and $\mathbf{F}$ is a field of subsets of U. By means of functions from $\mathbf{F}$ into $\{0,1\}$, there are obtained a finitely additive analogue of a classical absolute continuity partitioning theorem, a characterization of uniform absolute continuity, and an elementary argument for a theorem equivalent to a uniform absolute continuity theorem of Brooks and Dinculeanu (J. Math. Anal. Appl. **15** (1974), 156-175.)

1. Introduction. Let us begin by stating a well-known absolute continuity partitioning theorem.

THEOREM. *Suppose that U is a set*, S is a *σ-field of subsets of* U, *each of μ and ξ is a real nonnegative-valued countably additive measure on* S, *and ξ is absolutely continuous with respect to μ. Then there is a sequence $\{V_k\}_{k=1}^{\infty}$ of mutually exclusive sets of S such that $\cup_{k=1}^{\infty} V_k = U$ and such that if n is a positive integer*, W *is in S and $W \subseteq V_n$, then* $(n-1)\mu(W) \le \xi(W) \le n\mu(W)$.

The heuristic notion that underlies much of this paper is that, for the finitely additive case, zero-one set functions bear strong similarity to the elements of a σ-field.

We give the basic setting of this paper and then state the first of our three main theorems. It is a finitely additive analogue of the above theorem.

Suppose that U is a set, $\mathbf{F}$ is a field of subsets of U, $ba(\mathbf{F})$ is the set of all real-valued bounded finitely additive functions defined on $\mathbf{F}$, and, for each μ in $ba(\mathbf{F})$, A_μ denotes the set of all elements of $ba(\mathbf{F})$

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