ROCKY MOUNTAIN
JOURNAL OF MATHEMATICS
Volume 20, Number 4, Fall 1990

COEXISTENCE IN CHEMOSTAT-LIKE MODELS

PAUL WALTMAN

Dedicated to the memory of Geoffrey Butler

1. Introduction. Competition modeling is one of the more challenging aspects of mathematical biology. Competition is clearly important in nature yet there are so many ways for populations to compete that the modeling problem is difficult to do in any generality. On the other hand, the mathematical idea seems quite simple—when any population increases, the growth rate of the others should diminish, a concept that is quite easily expressed by partial derivatives of the specific growth rates. If an ecosystem is modeled by a system of differential equations, for example, by

$$y_i' = y_i f_i(y),$$

where $i = 1, 2, \dots, n, f_i$ is a nonnegative, continuously differentiable function defined on $\mathbf{R}^n$, and $y = (y_1, y_2, \dots, y_n)$, then competition is expressed by the condition

$$\frac{\partial f_i}{\partial y_j} \le 0$$

when $i \ne j$. Dynamical systems with such properties have been studied extensively, see Hirsch [**20, 21**] and Smith [**34**]. When $n = 2$, such dynamical systems preserve an order (leave a cone invariant) under the flow in forward time, a property which can yield valuable information about potential asymptotic behavior.

Such models easily reflect the direct impact of one population upon the other; for example, one produces metabolic products that inhibit the growth of the other. The simplest form of competition is where two or more populations compete for the same resource, for example, the same food supply or the same growth limiting nutrient. One

Research supported by NSF grant DMS 86-01398.