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SPHERICALLY SYMMETRIC SOLUTIONS OF AN ELLIPTIC-PARABOLIC NEUMANN PROBLEM

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1. Introduction. In [**4**] we gave an existence theorem for bounded weak solutions of the following problem:

$$(1.1) \qquad (c(u))_t = \Delta u \quad \text{in } Q_T = \Omega \times (0,T]$$

$$(1.2) \qquad (\mathbf{N}) \quad \frac{\partial u}{\partial \nu} = f \geq 0 \quad \text{on } \partial\Omega \times (0,T]$$

$$(1.3) \qquad c(u(x,0)) = v_0(x), \qquad x \in \Omega.$$

Here Ω is a bounded domain in $\mathbf{R}^n$ with smooth boundary $\partial\Omega$, $u = u(x,t)$ is the unknown function to be found and f and v_0 are given boundary values. Note that we prescribe the outward normal derivative $\partial u/\partial \nu$ at the lateral boundary of Q_T.

The function $c : \mathbf{R} \to \mathbf{R}$ is also given and it is assumed to be increasing on $\mathbf{R}^-$ and identically equal to one on $\mathbf{R}^+$, see Figure 1. Leaving smoothness assumptions aside for the moment, we recall that (1.1) reduces to

$$(1.4) \qquad \Delta u = 0 \text{ (elliptic)} \quad \text{for } u > 0$$

whereas

$$(1.5) \qquad u_t = \left\{\frac{1}{c'(u)}\right\} \Delta u \text{ (parabolic)} \quad \text{for } u < 0.$$

The physical background of (1.1) lies in the theory of partially saturated flows in porous media. In that context u stands for the hydrostatic potential due to capillary suction and $c(u)$ for the moisture content or saturation. The part of Ω where u is negative is the unsaturated region, and that where u is positive the saturated region. The set where $u = 0$ is usually referred to as the interface or free

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