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A NOTE ON FINITE LOCAL RINGS

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1. The aim of this short note is to correct an error concerning finite rings which has found its way into the literature and, so far as can be discovered, has not yet been corrected. Throughout, all rings are associative with unity (preserved by homomorphisms and inherited by subrings and overrings), and all unexplained terminology is standard.

2. The problem concerns an extension $R \subseteq S$ of commutative rings, in the case where S is a *separable* or a *Galois* extension of R. Suppose that S and R are commutative local finite rings; then it is stated at [**3**, Theorem 5.3] that S is a *separable* extension of R (see Paragraph 4) if and only if S is a *Galois* extension (see Paragraph 4 again) of R. Since a Galois extension is always a separable extension for any extension of commutative rings (Paragraph 4), the effect of this statement is that, for any separable extension $R \subseteq S$ of finite commutative local rings, S is free as an R-module (see [**3**, p. 532] and Paragraph 4 below). We give a counter-example to this statement in Paragraph 6 below and examine the reasons for the error in Paragraphs 7, 8.

3. A differently formulated—but equivalent—incorrect statement is given in [**4**, Theorem XIV], and, consequently, the implication "separable implies Galois" is false in [**4**, Corollary XV.3 and Theorem XV.11].

4. The basic facts about *separable* and *Galois* extensions of (general) commutative rings can be found in [**2**]; we note that S is *separable over* R if S is a projective (right) module over the enveloping algebra $S^e = S^{\mathrm{op}} \otimes_R S$ [**2**, p. 40], and that S is a *Galois* extension of R if it is a separable extension of R and if extra conditions identified at [**2**, pp. 80–81] are also satisfied. When S, R are finite and local (so that 0, 1 are the only idempotents of S, and their residue fields are Galois

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