

ON SUBSOCLES OF PRIMARY ABELIAN GROUPS

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ABSTRACT. The study of subsocles is an important part of the theory of primary abelian groups. In [5], section 66, closed, dense and discrete subsocles are defined in terms of the p -adic topology and some useful results about them are given. In this article we consider open subsocles and show that they too have interesting properties. We introduce the notion of range of such subsocles and establish various facts about this concept. We close the article with a characterization of subsocles of a given range using a new and natural generalization of the notion of purity.

All groups considered are abelian primary groups for a fixed prime number p . The terminology and notation not specifically explained here can be found in [5].

1. Open subsocles and their range. Let S be a subsocle of a p -group G . We say that S is an *open* subsocle of G if there exists a nonnegative integer n such that $p^n G[p] \subset S$. Such subsocle is open in the topology induced by the p -adic topology of G on $G[p]$. Open subsocles admit a large number of characterizations, some of which are collected in the following:

Theorem 1.1. *Let S be a subsocle of a p -group G . The following properties are equivalent*

- a) S is an open subsocle of G .
- b) $S \supset G^1[p] = (\cap p^n G)[p]$, and every pure subgroup of G containing S is a summand of G .
- c) G/K is bounded for every pure subgroup K of G containing S .
- d) G/K is reduced for every pure subgroup K of G containing S .
- e) S supports a pure subgroup K of G such that G/K is bounded.

The proof of Theorem 1.1 can be deduced from results in [1, 2].

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