ROCKY MOUNTAIN
JOURNAL OF MATHEMATICS
Volume 23, Number 2, Spring 1993

# ANALYTIC CONTINUATION OF FUNCTIONS GIVEN BY CONTINUED FRACTIONS, REVISITED

LISA LORENTZEN

ABSTRACT. Let the continued fraction $K(a_n(z)/b_n(z))$ converge to a meromorphic function $f(z)$ in a domain $D$. This function $f(z)$ may very well be meromorphic in a larger domain $D_0^*$ containing $D$, even if the continued fraction itself diverges in $D_0^*\backslash D$. The value of $f(z)$ for points $z \in D_0^*\backslash D$ can then be obtained by using modified approximants. This technique is known. We shall give sufficient conditions for obtaining a continuous extension of $f(z)$ to the boundary of $D_0^*$ by the same method.

**1. Introduction.** Let us illustrate the idea with some very simple examples which are taken from Thron and Waadeland's paper [**11**].

**Example 1.1.** The periodic, regular $C$-fraction

$$K(az/1) = \frac{az}{1} \, {}_{+} \, \frac{az}{1} \, {}_{+} \, \frac{az}{1} \, {}_{+} \cdots; \qquad a \in \mathbf{C}\backslash\{0\} \tag{1.1}$$

converges in the cut plane $D := \{z \in \mathbf{C}; |\arg(1+4az)| < \pi\}$ to the holomorphic function

$$w(z) := \frac{1}{2}((1+4az)^{1/2} - 1) \qquad \text{for } z \in D. \tag{1.2}$$

Clearly, $\Re((1+4az)^{1/2}) > 0$ for $z \in D$ in $w(z)$, and clearly, $w(z)$ can be extended analytically to the function

$$W(z) := \frac{1}{2}((1+4az)^{1/2} - 1) \qquad \text{for } z \in D^*, \tag{1.3}$$

where $D^*$ is the two-sheeted Riemann surface on which $W(z)$ is holomorphic. (That is, $D^*$ is a Riemann surface with branch points of

Received by the editors on August 31, 1989, and in revised form on September 10, 1991.
The author has changed her name from Lisa Jacobsen.