ROCKY MOUNTAIN
JOURNAL OF MATHEMATICS
Volume 23, Number 4, Fall 1993

A NEGATIVE ANSWER TO A QUESTION OF TECK-CHEONG LIM ABOUT PSEUDO CONVERGENCE

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A bounded sequence (x_n) in a Banach space X is said to *pseudo-converge* to a point x_0, called a pseudo limit, if x_0 minimizes the function

$$f_s(x) = \limsup_m ||y_m - x||$$

for every subsequence $S = (y_n)$ of (x_n). In the recent paper [**2**] the author put the following question: *Is it true that in a general Banach space X, if (x_n) pseudo-converges to θ, then there exist a sequence (z_n) in X and a sequence (z_n^*) in X^* such that $z_n^* \in J(z_n)$ for all $n \in \mathbf{N}$, $z_n^* \overset{w^*}{\to} \theta$ and $\lim_n ||x_n - z_n|| = 0$?* (here J denotes a duality map; see [**2**]).

In this short note we want to show that when $X = l_\infty$ the answer to the above question is negative. Let us choose $x_n' = e_n$, the unit vector basis of c_0; it is well known that it converges weakly to θ. Furthermore, it is simple to see that it pseudo-converges to more than one point of l_∞ ([**2**]). Choose one of its nonzero pseudo-limits x_0' and put $x_n = x_n' - x_0'$ for all $n \in \mathbf{N}$. Let us assume that there exist (z_n) and (z_n^*) as in the question above. It is clear that $z_n \overset{w}{\to} -x_0'$, too. Furthermore, $z_n^* \overset{w}{\to} \theta$ in $(l_\infty)^*$ (see [**1**, p. 103, Theorem 15]). Hence $(z_n^*, z_n) = (z_n^*, z_n + x_0') + (z_n^*, -x_0')$; using well-known results about $C(K)$ spaces (l_∞ is isomorphic to $C(\beta N)$!) (see [**1**, p. 113, Exercise 1]) we obtain that $(z_n^*, z_n) \to 0$ and so $z_n \overset{s}{\to} \theta$; this easily implies that $x_0' = \theta$. This contradiction concludes the proof.

At the end we observe that each space X with the Dunford-Pettis property and the Grothendieck property, too, can be used to answer in the negative Lim's question as done above (for these definitions and

Received by the editors on July 6, 1992.
Work performed under the auspices of G.N.A.F.A. of C.N.R. and partially supported by M.U.R.S.T. of Italy (40%–1990).
Key words. Pseudo-convergence, spaces of bounded sequences.