

ON THE NONEXISTENCE OF POSITIVE SOLUTIONS OF INTEGRO-DIFFERENTIAL INEQUALITIES

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ABSTRACT. In this paper we show that, under suitable conditions on f and K , the inequalities

$$-\lambda + \theta \int_0^{\infty} e^{\lambda s} K(s) ds > 0 \quad \text{for all } \lambda > 0$$

and

$$-\lambda^2 + \theta \int_0^{\infty} e^{\lambda s} K(s) ds > 0 \quad \text{for all } \lambda > 0$$

imply the integro-differential inequalities

$$y'(t) + \int_0^t K(t-s)f(y(s)) ds \leq 0 \quad \text{on } [T, \infty)$$

and

$$y''(t) - \int_0^t K(t-s)f(y(s)) ds \geq 0 \quad \text{on } [T, \infty)$$

have no positive solution, respectively, where $f(y)/y \geq \theta > 0$ in some interval $(0, y_0)$. We also point out that the function f cannot be a superlinear function, that is, $f(y) \neq y^\beta$ for $\beta \in (1, \infty)$.

1. Introduction. In this paper we consider the nonexistence of positive solutions of the following integro-differential inequalities

$$(E_1) \quad y'(t) + \int_0^t K(t-s)f(y(s)) ds \leq 0,$$

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