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THE EXTERIOR PROBLEM IN THE PLANE FOR A NON-NEWTONIAN INCOMPRESSIBLE BIPOLAR VISCOUS FLUID

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0. Introduction. The problem of steady flow of an incompressible fluid past a fixed body Ω' in R^2, as modeled by the Navier-Stokes system, has been extensively studied but is still mostly unresolved.

Using the Navier-Stokes equations to model this flow gives rise to the system:

$$\Delta \mathbf{v} - \mathbf{v}\nabla\mathbf{v} - \nabla p = 0 \tag{0.1}$$

$$\nabla \cdot \mathbf{v} = 0 \tag{0.2}$$

$$\mathbf{v} = 0 \quad \text{on } \partial\Omega' \tag{0.3}$$

$$\lim_{|x|\to\infty} \mathbf{v}(x) = \mathbf{v}_\infty \tag{0.4}$$

Léray has proved in 1933 (see [**12**]) the existence of a function which satisfies in a weak sense (0.1), (0.2) and (0.3). However, it is not known yet whether or not the weak solution constructed by Léray satisfies the boundary condition (0.4) (see [**9**] for example). R. Finn and D. Smith in [**3**] have resolved the situation under the additional condition that $|\mathbf{v}_\infty|$ be small enough. However, to this day and in spite of much talent and effort devoted to it, the question of existence of a solution to problem (0.1)–(0.4) is still open. For a more complete bibliography as well as a discussion of the different results, see the review paper by J. Heywood [**9**] and the references therein.

The Navier-Stokes model of fluid flow is based on the Stokes-hypothesis, which simplifies and restricts the relation between the stress tensor and the velocity. By relaxing the constraints of the Stokes hypothesis, the mathematical theory of multipolar viscous fluids generalizes the usual Navier-Stokes model in three important respects: it allows for nonlinear constitutive relations between the viscous part of

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