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# $Q$-REFLEXIVE BANACH SPACES

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Let $E$ be a Banach space. There are several natural ways in which any polynomial $P \in \mathcal{P}(^nE)$ can be extended to $\tilde{P} \in \mathcal{P}(^nE'')$, in such a way that the extension mapping is continuous and linear (see, for example, [**6**]). Taking the double transpose of the extension mapping $P \to \tilde{P}$ yields a linear, continuous mapping from $\mathcal{P}(^nE)''$ into $\mathcal{P}(^nE'')''$. Further, since $\mathcal{P}(^nE'')$ is a dual space, it follows that there is a natural projection of $\mathcal{P}(^nE'')''$ onto $\mathcal{P}(^nE'')$, and thus we have a mapping of $\mathcal{P}(^nE)''$ into $\mathcal{P}(^nE'')$. If all polynomials on a Banach space $E$ are weakly continuous on bounded sets, then these mappings from $\mathcal{P}(^nE)''$ into $\mathcal{P}(^nE'')$ coincide and have a particularly simple description. We discuss this in some detail below.

In this article we restrict ourselves to the situation in which all polynomials on $E$ are weakly continuous on bounded sets, and we study when this mapping is an isomorphism. As we will see, if three "ingredients" are present, then the mapping will be an isomorphism: (1) $E''$ has the Radon-Nikodym property [**18**], (2) $E''$ has the approximation property [**30**], and (3) every polynomial on $E$ is weakly continuous on bounded sets. In addition, we will construct an example of a quasi-reflexive (nonreflexive) Banach space $E$ for which the extension mapping is an isomorphism.

It is well known that $\mathcal{P}(^nE)$ is isomorphic to $(\oplus_{n,s}E)'$, the dual of the $n$-fold *symmetric* tensor product of $E$ endowed with the projective topology. In fact, our results carry over to the space $(\otimes_n E)'$. However, since our interest is in polynomials and holomorphic functions on $E$, we have preferred to concentrate on symmetric tensor products.

By [**9**, Theorem 2.9] if all continuous polynomials are weakly continuous on bounded sets, then they are in fact uniformly weakly continuous on bounded sets and so have a unique extension to polynomials

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