THE GLOBAL STRUCTURE OF UNIFORMLY ASYMPTOTICALLY ZHUKOVSKIJ STABLE SYSTEMS

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ABSTRACT. In this paper, we prove that the omega limit set of a uniformly asymptotically Zhukovskij stable orbit of a flow defined on a locally compact metric space is a closed orbit or a fixed point, and also it is a uniform attractor. If each orbit of the flow is uniformly asymptotically Zhukovskij stable, we obtain the global structure of the system, and further, if the space is compact, then the sum of fixed points and closed orbits is finite.

1. Introduction. There are many types of stabilities in the mathematical and physical literature, see [13]. Among the most important ones, Lyapunov stability is rather restrictive since it is an isochronous correspondence of orbits, for example, even an anharmonic oscillator is unstable in this sense [6, p. 41]. In this paper we shall consider a relaxed concept of stability, i.e., Zhukovskij stability [7]. It implies that orbits should be close to each other in the phase space and also repeat the ‘tracer’ of each other with a certain time lag; obviously such a stability is a kind of phase stability. The problem of studying periodicity for a limit orbit is old, and the literature on the subject is enormous, see [2, 3, 8, 10–12] and references therein. In 1966, Sell [12] proves that a bounded phase asymptotically stable solution of an autonomous system approaches an asymptotically stable periodic solution. In [3] Cronin gives conditions to guarantee that Lagrange stable solutions of a differential system in $\mathbb{R}^n$ are phase asymptotically stable, in the sense of [12], and therefore limit to a phase asymptotically stable periodic solution. Later, Li and Muldowney [8] obtain further results and simpler criteria to ensure that Lagrange stable solutions have periodic orbits as their limit sets, especially they show in [8, Theorem 2.1] that the omega limit set of a Lagrange stable orbit $\Gamma^+(x)$