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TOPOLOGICAL TRANSITIVITY AND MIXING NOTIONS FOR GROUP ACTIONS

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ABSTRACT. This paper surveys six notions of dynamical transitivity and mixing, in the context of group actions on topological spaces. We discuss the relations between these notions, and the manner in which they are inherited by subgroups, by taking products, and when passing to the induced action on hyperspace, i.e., the space of compact subsets. The focus of the paper is on the fact that certain standard notions, which are equivalent in the classical theory of the dynamics of flows and the iteration of single maps, are distinct for general group actions. The paper examines how the notions coalesce (a) for actions of abelian groups and (b) for chaotic actions.

0. Introduction. Consider an action of an infinite group G on a Hausdorff topological space M. This paper surveys six notions of dynamical transitivity and mixing for the action of G. We don't assume any particular topology on G, but we assume that the action is "continuous" in the sense that, for each group element g, the corresponding map $g : M \to M$ is a homeomorphism.

The fundamental transitivity and mixing notions are:

Definition 1. The action of G on M is:

(a) *topologically transitive* if, for every pair of nonempty open subsets U and V of M, there is an element $g \in G$ such that $gU \cap V \neq \varnothing$.

(b) *strongly topologically mixing* if for any pair of nonempty open subsets U and V of M, the set $\{g \in G;\ gU \cap V = \varnothing\}$ is finite.

(c) *topologically k-transitive for* $k \in \mathbf{N}$, if the induced action of G on the k-fold Cartesian product M^k is topologically transitive. Topological 2-transitivity is also called *weak topological mixing*.

Key words and phrases. Group action, mixing, topological transitivity.
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