

## POSITIVE SOLUTIONS OF SUPERLINEAR HAMMERSTEIN INTEGRAL EQUATIONS IN BANACH SPACES

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**ABSTRACT.** Some existence results of positive solutions for superlinear Hammerstein integral equations in Banach spaces are obtained by means of the fixed point index theory. Some applications to superlinear Sturm-Liouville problems in Banach spaces are given. We do not use the method of prior estimate which is used in many papers for similar problems. Our theorems extend some former results in this field.

**1. Introduction.** Let  $E$  be a Banach space. Consider the following Hammerstein integral equation in  $E$ :

$$(1) \quad \varphi(t) = \int_0^1 k(t, s) f(s, \varphi(s)) ds \equiv A\varphi(t),$$

where  $I = [0, 1]$ ,  $k(t, s) \in C[I \times I, R^1]$  is nonnegative,  $f \in C[I \times E, E]$  and  $\varphi \in C[I, E]$ . In this paper we get some existence theorems of positive solutions of equation (1) by means of the fixed point index theory and then give some applications to superlinear Sturm-Liouville problems in Banach spaces.

Many authors have studied the existence of nontrivial solutions for two-point boundary value problems (BVPs) of ordinary differential equations, cf. [5, 8, 3, 6]. In [5, 8, 3], the authors investigated the superlinear Sturm-Liouville problems. In [5], the superlinear condition is expressed as

$$\lim_{\substack{u \in P \\ \|u\| \rightarrow +\infty}} \frac{\phi(f(t, u))}{\phi(u)} = +\infty$$

uniformly in  $t \in [\alpha, \beta] \subset (0, 1)$ ,

where  $P$  is a cone of  $E$  and  $\phi$  is a positive linear functional. Clearly this superlinear condition is not very sharp since the righthand side of

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