JOURNAL OF INTEGRAL EQUATIONS
AND APPLICATIONS
Volume 9, Number 4, Fall 1997

A REPRESENTATION FORMULA FOR STRONGLY CONTINUOUS RESOLVENT FAMILIES

CARLOS LIZAMA

ABSTRACT. We give a representation formula for exponentially bounded strongly continuous resolvent families associated to an abstract Volterra equation of scalar type. As an application we derive a characterization of positive resolvent families defined in an ordered Banach space.

1. Introduction. We consider the following Volterra equation defined on a complex Banach space X

$$u(t) = f(t) + \int_0^t a(t-s)Au(s)\,ds, \quad t \in J \tag{1.1}$$

where A is a closed linear unbounded operator in X with dense domain $D(A)$, $a \in L^1_{\text{loc}}(\mathbf{R}_+)$ is a scalar kernel $\neq 0$ and $f \in C(J, X)$, $J := [0, T]$.

The basic concept concerning (1.1) is that of well-posedness which is the direct extension of the corresponding notion usually employed for the abstract Cauchy problem (of first order)

$$\dot{u}(t) = Au(t), \qquad u(0) = u_0. \tag{1.2}$$

It is well known that well-posedness is equivalent to the existence of a resolvent $\{S(t)\}_{t\geq 0} \subseteq \mathcal{B}(X)$ for (1.1), i.e., a strongly continuous family of bounded linear operators in X which commutes with A and satisfies the resolvent equation

$$S(t)x = x + \int_0^t a(t-s)AS(s)x\,ds,$$
$$t \geq 0, \quad x \in D(A).$$

Received by the editors on August 20, 1996, and in revised form on March 10, 1997.
This research was done while the author was visiting the Mathematisches Institut, Universität Tübingen, supported by a grant of the Alexander von Humboldt Foundation.