JOURNAL OF INTEGRAL EQUATIONS
AND APPLICATIONS
Volume 9, Number 3, Summer 1997

THE SOLUTION IN A CLASS OF SINGULAR FUNCTIONS OF CAUCHY TYPE BISINGULAR INTEGRAL EQUATIONS

D. ELLIOTT, I.K. LIFANOV AND G.S. LITVINCHUK

ABSTRACT. We first give the solution, in a class of singular functions, of a one-dimensional singular integral equation with Cauchy integral, arising from a problem in two-dimensional aerodynamics. The analysis is then extended to the solution of a bisingular Cauchy singular integral equation arising from three-dimensional airfoil theory.

1. Introduction. Traditionally, the theory of singular integral equations with Cauchy kernel has been constructed under the assumption that the unknown function is absolutely integrable. See, for example, the texts by Muskhelishvili [**7**] or Gakhov [**4**]. Such theory has immediate application in the mathematical theories of elasticity and hydrodynamics. But airfoil control problems lead to the necessity of introducing an ejection external stream emanating from a point or a line on the airfoil which can be modeled by the introduction of a source into the airfoil; see Belotserkovsky and Lifanov [**2**] and Woolard [**9**]. In the simplest case, this modeling leads to the necessity of solving a singular integral equation with Cauchy kernel on an interval, with the solution having a non-integrable singularity of the form $1/x$ near $x = 0$. The so-called method of discrete vortices has been applied by Lifanov, Mikhailov and Titsky [**6**] for the case of incompressible flow past an airfoil, with a slit in the airfoil possessing a source. An explicit analytic solution to this problem is not known but Bisplinghoff, Ashley and Halfman [**3**] have shown that in the case of a rectangular wing, the problem is reduced to that of solving a bisingular integral equation of the first kind with Cauchy integrals taken over the Cartesian product of two finite intervals. We shall show that, in a special case, it is possible to write down explicitly all the solutions that are absolutely integrable. In addition, we shall also give analytic solutions of this equation having

Received by the editors on June 1, 1996, and in revised form on April 8, 1997.
This project has been partially supported by a grant from the Australian Research Council.