

**EXISTENCE AND UNIQUENESS FOR SPATIALLY  
INHOMOGENEOUS COAGULATION-CONDENSATION  
EQUATION WITH UNBOUNDED KERNELS**

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**ABSTRACT.** We prove the existence and uniqueness theorem for Smoluchovsky's model with condensation in three-dimensional space. Initial data are supposed to be small enough.

**1. Introduction.** We are concerned with the space-inhomogeneous Smoluchovsky's equation with condensation processes taken into account.

$$(1.1) \quad \begin{aligned} & \frac{\partial}{\partial t} c(x, z, t) + \frac{\partial}{\partial x} (r(x)c(x, z, t)) + \operatorname{div}_z (v(x, z)c(x, z, t)) \\ &= \frac{1}{2} \int_0^x K(x-y, y)c(x-y, z, t)c(y, z, t) dy \\ & \quad - c(x, z, t) \int_0^\infty K(x, y)c(y, z, t) dy; \\ & \quad x, t \in R_+^1 = [0, \infty), \quad z \in R^3. \end{aligned}$$

It describes the time evolution of particles in disperse systems. Symmetric nonnegative on  $R_+^2$  function  $K(x, y)$  define intensity of merging of particles with masses  $x$  and  $y$ . Unknown function  $c(x, z, t)$  is a distribution function for particles of the disperse system of mass  $x \geq 0$  at time  $t \in R_+^1 = [0, \infty)$  at space point  $z \in R^3$ . The function  $v(x, z)$  is a velocity of space transfer of particles;  $r(x)$  is a scalar speed of growth of particles due to condensation of molecules (or, more generally, clusters) from outer medium, e.g., condensation of vapor on water drops in atmospheric clouds. In physically real situations we often have  $r \sim x^\alpha$ ,  $\alpha > 0$ ,  $0 < x_0 \leq x \leq \bar{x}$ , where  $x_0$  is a critical mass of a particle which splits regions of its stable and unstable state;  $\bar{x}$  is a conventional

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