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ROTHE'S METHOD FOR THE HEAT EQUATION AND BOUNDARY INTEGRAL EQUATIONS

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Dedicated to E. Martensen on the occasion of this 70th birthday

ABSTRACT. Rothe's method for parabolic initial boundary value problems, also known as the horizontal line method, consists of a time discretization by finite differences and leads to a sequence of boundary value problems for an inhomogeneous elliptic equation. Whereas in the traditional approach in the solution of this sequence of boundary value problems volume potentials are incorporated, in order to preserve the advantages of the boundary integral equation method we present an approach involving only boundary integrals.

1. Introduction. For boundary value problems for elliptic differential equations with constant coefficients the use of boundary integral equations has a long history both for establishing existence of solutions and for numerical approximations. Similarly, integral equations have also been successfully applied to initial boundary value problems for parabolic equations. These integral equations, in general, are of Fredholm type with respect to the space variable and of Volterra type with respect to the time variable. Due to the simplicity of the Fredholm alternative and the use of successive approximations, integral equations of the second kind for the heat equation have been considered already for almost a century. Following earlier work of Holmgren [**11, 12**] and Gevrey [**10**], a rigorous existence proof for the initial boundary value problem with Dirichlet boundary condition via an integral equation of the second kind obtained through a double-layer heat potential approach was given by Müntz [**20**] already in 1934 (see also [**14**]). On the other hand, a corresponding theory for the integral equation of the first kind arising from a single-layer heat potential approach has been developed only very recently by Arnold and Noon [**1**], by Costabel [**6**] and by Hsiao and Saranen [**13**].

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