JOURNAL OF INTEGRAL EQUATIONS
AND APPLICATIONS
Volume 9, Number 1, Winter 1997

UNIFORM CONVERGENCE ESTIMATES FOR A COLLOCATION METHOD FOR THE CAUCHY SINGULAR INTEGRAL EQUATION

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ABSTRACT. The authors study the convergence and the stability of a collocation and a discrete collocation method for Cauchy singular integral equations with weakly singular perturbation kernels in some weighted uniform norms. Uniform error estimates are also given.

1. Introduction. We consider the Cauchy singular integral equation with constant coefficients

$$(1.1)\quad au(x)+\frac{b}{\pi}\int_{-1}^{1}\frac{u(t)}{t-x}\,dt+\int_{-1}^{1}k(x,t)u(t)\,dt=f(x),\qquad |x|<1,$$

where the first integral in (1.1) is to be understood in the sense of Cauchy principal value. Here u is the unknown solution, a and b are given real constants such that $a^2+b^2=1$, $b\neq 0$, f is a Hölder continuous function, and k is a weakly singular function of the form

$$(1.2)\qquad k(x,t)=\frac{H(x,t)}{|t-x|^{\mu}},\qquad 0<\mu<1,$$

with $H(x,t)\in \mathrm{Lip}_{\nu}([-1,1]^2)$, $0<\nu\leq 1$. Here $\mathrm{Lip}_{\nu}(A)$ is the space defined by

$$\mathrm{Lip}_{\nu}(A):=\left\{g\in C^0(A):\sup_{x\neq y\in A}\frac{|g(x)-g(y)|}{|x-y|^{\nu}}:=M_g<\infty\right\}$$

and equipped with the norm

$$\|g\|_{\nu}:=\|g\|+M_g,\quad \text{where}\quad \|g\|=\max_{x\in A}|g(x)|.$$

Received by the editors on April 30, 1996, and in revised form on November 15, 1996.