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NUMERICAL SOLUTION OF PERIODIC FREDHOLM INTEGRAL EQUATIONS OF THE SECOND KIND BY MEANS OF ATTENUATION FACTORS

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ABSTRACT. We present a general method for solving linear periodic Fredholm integral equations of the second kind. The method is based on attenuation factors and makes use of the fast Fourier transform (FFT). It can be applied to a large class of approximation schemes such as spline interpolants or smoothing processes. We add some convergence results as well as an iterative method for the solution of the system of linear equations arising from the discretization.

1. Introduction. Let I be the interval $[0, 2\pi] \subset \mathbf{R}$ and I^2 the square $[0, 2\pi] \times [0, 2\pi] \subset \mathbf{R}^2$. Let $L_2 \equiv L_2(I)$ denote the complex Hilbert space of square integrable functions and $L_2(I^2)$ the corresponding bivariate space. For a Hilbert-Schmidt kernel h, i.e., a function $h(t,s) \in L_2(I^2)$, consider the bounded, linear and compact operator $(\mathrm{Int}\, h)$ defined by

$$(\mathrm{Int}\, h) : f \in L_2 \longrightarrow (\mathrm{Int}\, h) f := \frac{1}{2\pi} \int_0^{2\pi} h(\cdot, s) f(s)\, ds \in L_2.$$

Such an operator is called a Hilbert-Schmidt integral operator (H-S operator) and the equation

$$x + (\mathrm{Int}\, h) x = f, \tag{1.1}$$

where the righthand side function f belongs to L_2, is a Fredholm integral equation of the second kind in L_2 for the unknown function $x \in L_2$. Setting $H := (\mathrm{Int}\, h)$, (1.1) can be written as

$$x + Hx = f. \tag{1.2}$$

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