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ON THE INVERSE OF INTEGRAL OPERATORS WITH KERNEL OPERATORS

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ABSTRACT. We study the boundedness of integral operators whose kernels are functions of operators $Vf(x) := f(x) + \int k(x,t,L)f(t)\,d\mu(t)$, where $k(x,t,\lambda)$ is an entire function of λ and L is an unbounded self-adjoint operator in $L^2_{d\mu(t)}$. By using Korotkov's theorem we derive a simple necessary condition for V to be a Carleman type operator. We are particularly interested in the cases when the inverse operator exists and has the same form as V. This study provides a new method for the inversion of integral equation of Carleman type.

1. Introduction. We first are interested in the boundedness of operators defined by

$$Vf(x) = f(x) + \int k(x,t,L)f(t)\,d\mu(t), \quad d\mu(x) \text{ a.e.}$$

in the Hilbert space $L^2_{d\mu(x)}$, where $k(x,t,\lambda)$ is an entire function of λ, $d\mu$ measurable in x and t, and L is an unbounded self-adjoint operator acting in the separable Hilbert space $L^2_{d\mu(t)}$. In fact one needs $k(x,t,\lambda)$ to be an analytic function of λ in the neighborhood of the spectrum of L only.

For the sake of simplicity we shall assume that

$$k(x,t,\lambda) := \sum_{n\geq 0} a_n(x,t)\lambda^n$$

and so $Vf(x)$ is defined by

$$Vf(x) := f(x) + \int \sum_{n\geq 0} a_n(x,t)L^n f(t)\,d\mu(t), \quad d\mu(x) \text{ a.e.} \tag{1}$$

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