JOURNAL OF INTEGRAL EQUATIONS
AND APPLICATIONS
Volume 7, Number 2, Spring 1995

ACCURATE COMPUTATION OF THE FIELD IN PIPPARD'S NONLOCAL SUPERCONDUCTIVITY MODEL

TAO LIN AND ROBERT C. ROGERS

ABSTRACT. A Galerkin finite element method is analyzed for a class of the Fredholm type integro-differential equations. The method is applied to Pippard's nonlocal superconductivity model. Optimal H^1 norm error estimates are derived for the finite element solution of the current potential. A class of superconvergent post-processing techniques are developed to obtain more accurate approximations to the magnetic field from the finite element solutions. An H^1 semi-norm actual error indicator is derived and is used to generate an adaptive grid refinement procedure. Several numerical examples are presented.

1. Introduction. The purpose of this paper is to develop and analyze the Galerkin finite element solution of the following boundary value problem of a Fredholm type integro-differential equation:

$$Du(x) + \int_{-l}^{l} K(x,y)u(y)\,dy = f(x), \qquad x \in (-l,l), \tag{1.1}$$

$$a_2(-l)u'(-l) = b_0, \qquad a_2(l)u'(l) = b_1. \tag{1.2}$$

We assume throughout this paper that the kernel $K : [-l,l] \times [-l,l] \to \mathbf{R}$ is such that the mapping $\mathcal{K} : L^2(-l,l) \to L^2(-l,l)$ defined by

$$L^2(-l,l) \ni u \mapsto \mathcal{K}\,u(\cdot) := \int_{-l}^{l} K(\cdot,y)u(y)\,dy \in L^2(-l,l) \tag{1.3}$$

Received by the editors on May 10, 1994, and in revised form on September 1, 1994.
The research of the first author was supported in part by grant DMS-9403844 from the National Science Foundation.
The research of the second author was supported in part by grants DMS-9204304 and DMS-9403844 from the National Science Foundation.