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DIRICHLET CONVOLUTION INVERSES AND SOLUTION OF INTEGRAL EQUATIONS

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ABSTRACT. A method for the solution of integral equations based on the concepts of Dirichlet convolutions and convolution inverses is presented. This method generalizes the procedures of Chen.

1. Introduction. Recently, Chen [**2, 3**] gave the solution of the integral equations for the photon density of states and for the inverse blackbody radiation problem for remote sensing by using an inversion formula from number theory. His work generated some interest, particularly in the possible applications of number theory in physics [**7, 8**].

Our present aim is to give a general procedure for the solution of certain classes of integral equations of the first kind based on the inversion of the Dirichlet convolutions, a subject studied in elementary number theory [**1**]. Chen's method becomes an interesting particular case.

Interestingly, the asymptotic behavior of series of the type

$$\sum_{n=1}^{\infty} a_n \phi(n\varepsilon)$$

as $\varepsilon \to 0^+$ was studied by using the theory of distributions [**4, 5, 6**]. These series play an important role in the method presented here. In [**4**] many results of number theoretical importance are obtained by using distributions, an old acquaintance of physicists, who used them before mathematicians.

2. Dirichlet multiplication and inversion. The concepts of Dirichlet multiplication and inversion, expounded below, are well known. Details can be found in standard texts in number theory [**1**].

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