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A TIME DEPENDENT PARABOLIC INITIAL BOUNDARY VALUE DELAY PROBLEM

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1. Introduction. In this paper we use the theory of analytic semigroups in a Banach space to solve the following second order parabolic initial-boundary value problem with a discrete and a continuous delay term:

$$\begin{aligned} u_t &= \mathcal{A}(t,x)u(t,x) + \mathcal{A}(t,u)u(t-r,x) \\ &\quad + \int_{-r}^{0} a(\sigma)\mathcal{A}(t,x)u(t+\sigma,x)\,d\sigma \\ &\quad + f(t,x) \qquad \text{for } (t,x) \in Q_T \end{aligned}$$

$$\begin{aligned} u(t,x) &= k(t,x) \qquad \text{for } (t,x) \in [-r,0] \times \overline{\Omega} \\ \mathcal{B}(t,x)u(t,x) &= g(t,x) \qquad \text{for } (t,x) \in [-r,T] \times \Gamma \end{aligned} \tag{1.1}$$

where Ω is an open bounded set of R^n with a smooth boundary Γ; r and T are positive numbers, $Q_T = [0,T] \times \overline{\Omega}$ and f, k, g and a are functions belonging to suitable Banach spaces. The operator

$$\mathcal{A}(t,x) = \sum_{i,j=1}^{n} a_{ij}(t,x)D^{ij} + \sum_{i=1}^{n} b_i(t,x)D^i + cI, \tag{1.2}$$

for every $t \in [0,T]$ is elliptic, and the boundary operator

$$\mathcal{B}(t,x) = \sum_{i=1}^{h} \beta_i(t,x)D^i + \gamma(t,x)I \tag{1.3}$$

is nontangential.

First we study the autonomous case, i.e., the case where a_{ij}, b_i, c, β_i and γ do not depend on the variable t. We obtain a maximal regularity result in a suitable interval $[0,t_1]$ contained in $[0,r]$, then we repeat the

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