JOURNAL OF INTEGRAL EQUATIONS
AND APPLICATIONS
Volume 3, Number 4, Fall 1991

PARABOLIC INTEGRODIFFERENTIAL EQUATIONS WITH SINGULAR KERNELS

DANIELA SFORZA

ABSTRACT. We consider a parabolic integrodifferential Volterra equation with nonhomogeneous boundary condition

$$(*)\qquad \begin{cases} u_t(t,x) = (\Delta + c)\int_0^t k(t-s)u(s,x)\,ds + k_0 u(t,x) + f(t,x), \\ \qquad\qquad t \in [0,T], x \in \Omega, \\ u(0,x) = u_0(x), \quad x \in \Omega \\ u(t,x) = \varphi(t,x), \quad t \in [0,T],\ x \in \partial\Omega, \end{cases}$$

where Δ is the Laplace operator and k is a scalar kernel singular at $t = 0$. This assumption on k gives a parabolic character to $(*)$. We state some results about the existence, uniqueness and regularity of the solutions of $(*)$.

0. Introduction. This paper is concerned with a class of parabolic integrodifferential Volterra equations with nonhomogeneous boundary condition

(0.1)
$$\begin{cases} u_t(t,x) = (\Delta + c)\int_0^t k(t-s)u(s,x)\,ds + k_0 u(t,x) + f(t,x), \\ \qquad\qquad t \in [0,T],\ x \in \Omega, \\ u(0,x) = u_0(x), \quad x \in \Omega, \\ u(t,x) = \varphi(t,x), \quad t \in [0,T],\ x \in \partial\Omega, \end{cases}$$

where Ω is a bounded open set in $\mathbf{R}^n$, $n \in \mathbf{N}$, with regular boundary $\partial\Omega$, c and k_0 are real constants, Δ is the Laplace operator and the kernel k is a scalar real function.

Problem (0.1) occurs in the study of heat flow in materials with memory (see [**10, 13, 14**] and references therein).

In the applications one is often concerned with the corresponding problem with infinite delay (that is, with $\int_0^t$ replaced by $\int_{-\infty}^t$), which

AMS *Mathematics Subject Classification*. Primary 45K05, 34A10.
Key words and phrases. Volterra equation, Laplace transform, resolvent operator, Dirichlet map.