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# THE SOLUTION OF INTEGRAL EQUATIONS WITH DIFFERENCE KERNELS

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ABSTRACT. This paper investigates integral equations with difference kernels posed on finite intervals. Formulae relating the solutions of second kind equations corresponding to particular free terms, including one of the "imbedding" variety, are derived using straightforward operator manipulation. These lead to an explicit expression for the solution of the second kind equation with a general free term. Some attention is given to the practically important logarithmically singular kernel for both first and second kind equations.

**1. Introduction.** Suppose that the integral equation

$$\mu\phi(x) = f(x) + \int_{-\infty}^{\infty} k(x-t)\phi(t)\,dt, \quad -\infty < x < \infty, \tag{1.1}$$

has a solution $\phi$ for "suitable" given functions $f$ and $k$. It is not difficult to verify that this solution is given by

$$\mu\phi(x) = f(x) + \int_{-\infty}^{\infty} r(x-t)f(t)\,dt, \quad -\infty < x < \infty,$$

where $r$ satisfies the integral equation

$$\mu r(x) = k(x) + \int_{-\infty}^{\infty} k(x-t)r(t)\,dt, \quad -\infty < x < \infty.$$

This conclusion is valid if, for example, the functions involved are in $L_2(-\infty, \infty)$.

One of the objectives of this paper is to derive a corresponding result for the integral equation

$$\mu\phi(x) = f(x) + \int_0^1 k(x-t)\phi(t)\,dt, \quad 0 \le x \le 1. \tag{1.2}$$