JOURNAL OF INTEGRAL EQUATIONS
AND APPLICATIONS
Volume 3, Number 3, Summer 1991

ON THE THEORY OF PARTIAL INTEGRAL OPERATORS

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The fundamental work of F. Riesz and J. Schauder has shown that the basic facts of the classical theory of integral equations (convergence of iteration methods, Fredholm alternative, bilinear expansions of kernels, etc.) are due to certain functional-analytic and geometric properties of corresponding integral transforms, such as boundedness (continuity), compactness, or weak compactness. With regard to this fact, many authors tried to find conditions for the continuity or compactness of linear integral operators in various function spaces. Now the theory of such operators is rather advanced and complete; the basic results may be found, for example, in the monographs [**9, 20, 22, 25, 38**].

Unfortunately, the operators studied in these monographs do not cover many integral operators arising in mathematical physics. For instance, some problems for elliptic or hyperbolic equations lead to integral equations with the property that the integration is carried out only over some of the variables [**8, 30, 32**]; such equations will be called *partial integral equations* in what follows. For a long time, such equations have not been studied for essentially two reasons. First of all, partial integral equations occur less often than classical integral equations (involving integration with respect to all variables); second, the corresponding operators are not compact, and thus the classical Riesz-Schauder theory does not apply. In recent years, however, it became clear that partial integral equations should be investigated in more detail. In fact, they arise in many fields of current interest, especially in continuum mechanics. Here one could mention, for instance, axial-symmetric contact problems [**1–3, 23, 24, 28, 29**], the theory of thin elastic shells [**32**], and certain problems in aerodynamics [**4**].

It is clear that, for studying partial integral equations, one has to analyze the operators generated by such equations. We mention here the papers [**5–7, 12–19, 27**], where spectral properties of such

AMS *Mathematics Subject Classification.* Primary 45P05, 45A05; Secondary 47G05, 45D05, 46E30, 26B99.