JOURNAL OF INTEGRAL EQUATIONS
AND APPLICATIONS
Volume 11, Number 4, Winter 1999

ON THE EFFICIENT DISCRETIZATION OF INTEGRAL EQUATIONS OF THE THIRD KIND

SERGEI V. PEREVERZEV, EBERHARD SCHOCK
AND SERGEI G. SOLODKY

ABSTRACT. A new discretization scheme for solving ill-posed integral equations of the third kind is proposed. We show that when this scheme is combined with Morozov's discrepancy principle for Landweber iteration, the resulting method is more efficient than the collocation method, in terms of the order of the number of arithmetic operations required to achieve a given accuracy in the approximate solution.

1. Introduction. In his fundamental papers on integral equations, Hilbert [**5**] introduced the notion of integral equations of the first, second and of the third kind. A linear integral equation

$$rx + Kx \equiv r(t)x(t) + \int_0^1 k(t,\tau)x(\tau)\,d\tau = y(t) \tag{1}$$

is said to be of the first kind if $r \equiv 0$, of the second kind if r is a nonzero constant, and of the third kind if r is a function with zeros in its domain (if r is never zero the equation is equivalent to an equation of the second kind). If the function r is continuous and has a finite number of zeros, then equation (1) is a special type of nonelliptic singular integral equation investigated by Prössdorf [**11**]. For functions r with known zeros approximate methods for solving integral equation (1) were proposed by Gabbasov, see, for example, [**3**]. But these methods are completely unusable if r is, for example, a characteristic function of a proper subset of positive measure. Moreover, as indicated in [**12**], if for each neighborhood V of zero the inverse $r^{-1}(V)$ of V has positive measure, then the problem of solving the equation (1) is not well posed in the sense of Hadamard and therefore regularization techniques are required for solving (1). In our opinion it makes sense to apply the regularization methods, even when the function r has a finite number of zeros with unknown locations.

Accepted for publication on April 27, 1999.