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BOUNDED REPRESENTATION AND RADIAL PROJECTIONS OF BISECTORS IN NORMED SPACES

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ABSTRACT. It is well known that the description of topological and geometric properties of bisectors in normed spaces is a non-trivial subject. In this paper we introduce the concept of bounded representation of bisectors in finite-dimensional real Banach spaces. This useful notion combines the concepts of bisector and shadow boundary of the unit ball, both corresponding with the same spatial direction. The bounded representation visualizes the connection between the topology of bisectors and shadow boundaries (Proposition 2) and gives the possibility to simplify and to extend some known results on radial projections of bisectors. Our main result (Theorem 1) says that, in the manifold case, the topology of the closed bisector and the topology of its bounded representation are the same; they are closed, $(n-1)$-dimensional balls embedded in Euclidean n-space in the standard way.

1. Introduction and some preliminary results. In recent times, Minkowski geometry (i.e., the geometry of finite dimensional, real Banach spaces, see [**13**]) became again an important research field. Strongly related to Banach space theory, Finsler geometry, and classical convexity, it is permanently enriched by new results in applied disciplines. The most examined concepts of it naturally connect to physics, functional analysis, and non-Euclidean geometries. We will not introduce basic notions and terminology of this field going beyond our purpose; for its fundamentals the reader is referred to the monograph [**13**] and to the surveys [**10, 11**]. The present paper refers to *bisectors* in (finite dimensional normed or) *Minkowski spaces*, i.e., to collections of points which have, in each case, the same distance (with respect to the corresponding norm) to two given points $\mathbf{x}$, $\mathbf{y}$ of these spaces. Note that bisectors in Minkowski spaces play an essential role in discrete and

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