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CROSSED PRODUCTS OF NONCOMMUTATIVE CW-COMPLEXES BY FINITE GROUPS

ERICH MCALISTER

ABSTRACT. In this paper we will construct a new class of examples of the so-called noncommutative CW-complexes ($NCCW$-complexes). We show that if G is a finite group acting on a $NCCW$-complex $\mathbf{A}_n$ by a natural class of automorphisms, then the crossed product $\mathbf{A}_n \rtimes G$ is an $NCCW$-complex. As a result, we find that whenever G is a finite group of diffeomorphisms acting on a smooth manifold M, then the resulting crossed product $C(M) \rtimes G$ has the structure of an $NCCW$-complex. Partial results are given in the case of twisted crossed products.

1. Introduction. The goal of this paper is to give a systematic study of crossed products of the form $\mathbf{A} \rtimes G$, where G is a finite group and $\mathbf{A}$ can be decomposed in a way analogous to the cellular decomposition of a topological CW-complex as first defined in [**1**]. The results given provide the general framework for the computations of certain K-theories carried out in [**7**].

Motivations for this study come variously from [**9, 11, 12**]. First, crossed products resulting from the action of finite groups on simplicial complexes were studied at some length by Yang in [**12**]. The primary goal of this author was to compute the K-theory of group C^*-algebras of planar crystallographic groups. These algebras were realized as subhomogeneous algebras over simplicial complexes where dimension drops occur only on lower dimensional skeleta. While this realization did allow for computation of $K_*(C^*(G))$ for all 17 planar crystallographic groups, nontrivial analysis was required in the computations.

The motivation for studying these crossed products from the point of view of noncommutative CW-complexes can be summarized by the characterization of noncommutative CW-complexes as "algebras of matrix-valued functions over topological spaces homeomorphic to

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