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MULTIPLICITY OF SOLUTIONS FOR ELLIPTIC SYSTEMS WITH TOPOLOGICAL METHODS

YINGHUA JIN, Q-HEUNG CHOI AND KYEONGPYO CHOI

ABSTRACT. We study the existence and multiplicity of solutions of Dirichlet boundary value problem for nonlinear elliptic systems of the form $-\Delta u = f(x,u,v)$, $-\Delta v = g(x,u,v)$ in Ω, where Ω is a bounded open set in R^n with smooth boundary $\partial\Omega$. To study the system we use the topological methods.

1. Introduction. In applications of differential equations, critical points correspond to weak solutions of the equation. Many nonlinear problems in physical science can be reduced to finding critical points of the corresponding functionals. Indeed, this fact makes critical point theory which is an important existence tool in studying nonlinear differential equations.

The elliptic system has an extensive practical background. It can be used to describe the multiplicate chemical reaction catalyzed by catalyst grains under constant or variant temperature, and it can be a simple model of tubular chemical reaction; more naturally, it can be a correspondence of the stable station of a dynamical system determined by the reaction-diffusion system (see [**3**]).

The system of nonlinear elliptic equations present some new and interesting phenomena, which are not presented in the study of a single equation. In this paper we study existence and multiplicity of solutions for nonlinear elliptic systems of the form

$$\begin{aligned} -\Delta u &= f(x,u,v) \quad \text{in } \Omega, \\ -\Delta v &= g(x,u,v) \quad \text{in } \Omega, \end{aligned}$$

where $\Omega \subset R^n$ is a bounded smooth domain, subject to Dirichlet boundary conditions $u = v = 0$ on $\partial\Omega$. To study the system we use topological methods.

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