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FOURIER-FEYNMAN TRANSFORMS, CONVOLUTIONS AND FIRST VARIATIONS ON THE SPACE OF ABSTRACT WIENER SPACE VALUED CONTINUOUS FUNCTIONS

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ABSTRACT. In this paper, we establish various relationships among the Fourier-Feynman transform, convolution and first variation of functionals in some Banach algebra, defined on the space of abstract Wiener space valued continuous functions, which corresponds to the Banach algebra defined on classical Wiener space introduced by Cameron and Storvick.

1. Introduction. The concept of an L_1 analytic Fourier-Feynman transform for functionals on classical Wiener space was introduced by Brue [**2**]. In [**3**] Cameron and Storvick introduced an L_2 analytic Fourier-Feynman transform on classical Wiener space. In [**14**] Johnson and Skoug developed an L_p analytic Fourier-Feynman transform theory for $1 \leq p \leq 2$ which extended the results in [**2, 3**] and gave various relationships between the L_1 and the L_2 theories. In [**11, 12, 13**], Huffman, Park and Skoug defined a convolution product for functionals on classical Wiener space, and they showed that the analytic Fourier-Feynman transform of convolution product is the product of transforms. In [**18**], Park, Skoug and Storvick investigated various relationships among the first variation, the Fourier-Feynman transform and the convolution product for functionals on classical Wiener space that belong to the Banach algebra $\mathcal{S}$ introduced by Cameron and Storvick in [**4**]. Recently, Ahn, Chang, Kim, Song and Yoo studied the Fourier-Feynman transform theory on abstract Wiener space [**1, 7, 8, 9**]. For a detailed survey of this topic, see [**20**].

Let (H, B, ν) be an abstract Wiener space. Let $C_0(B) \equiv C_0([0,T], B)$ denote the space of abstract Wiener space valued continuous functions $x(t)$ which are defined on $[0,T]$ with $x(0) = 0$. From [**17**] it follows

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