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MULTIPLE DIRICHLET SERIES INTERPOLATING BELL NUMBERS AND STIRLING NUMBERS

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ABSTRACT. The number of ways that a set of n elements can be partitioned into nonempty subsets is called the nth Bell number. An interpolation Dirichlet series of Bell numbers is well-known classically. In this paper, as its generalization, we construct a certain multiple Dirichlet series which interpolates Bell numbers. As another example, we construct a multiple Dirichlet series which interpolates Stirling numbers of the second kind.

1. Introduction. Let $\mathbf{N}$ be the set of natural numbers, $\mathbf{N}_0 = \mathbf{N} \cup \{0\}$, $\mathbf{Z}$ the ring of rational integers, $\mathbf{Z}^* = \mathbf{Z} \setminus \{0\}$, $\mathbf{Q}$ the field of rational numbers, $\mathbf{R}$ the field of real numbers, and $\mathbf{C}$ the field of complex numbers.

The nth Bell number β_n is defined as the number of ways that a set of n elements can be partitioned into nonempty subsets, see [**2**]. Formally we let $\beta_0 = 1$. Then $\{\beta_n \mid n \in \mathbf{N}_0\}$ satisfy

$$\beta_n = \sum_{j=0}^{n-1} \binom{n-1}{j} \beta_j, \quad n \in \mathbf{N}.$$

By this relation, we can calculate that $\beta_1 = 1$, $\beta_2 = 2$, $\beta_3 = 5$, $\beta_4 = 15$, $\beta_5 = 52$, etc. In the twentieth century, various properties of $\{\beta_n\}$ were studied, for example,

$$e^{e^x - 1} = \sum_{n=0}^{\infty} \beta_n \frac{x^n}{n!}; \tag{1}$$

$$\frac{1}{e} \sum_{k=1}^{\infty} \frac{k^n}{k!} = \beta_n, \quad n \in \mathbf{N}, \tag{2}$$

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