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REARRANGEMENT INVARIANT SUBSPACES OF LORENTZ FUNCTION SPACES II

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ABSTRACT. For $1 \leq q \leq p < \infty$ and $p > 2$, it is shown that the only subspaces of the Lorentz function space $L_{p,q}[0,1]$ which are isomorphic to r.i. function spaces on $[0,1]$ are $L_2[0,1]$ and $L_{p,q}[0,1]$, up to equivalent renormings. If $p < 2$ or if $1 < p < q < \infty$, then $L_{p,q}[0,1]$ has an r.i. subspace which is not isomorphic to either $L_2[0,1]$ or $L_{p,q}[0,1]$.

1. Introduction. This note is an addendum to a previous paper by the author [5] in which it is shown that for $2 \leq q < p < \infty$ the only rearrangement invariant function spaces on $[0,1]$ that embed isomorphically into the Lorentz function space $L_{p,q} = L_{p,q}[0,1]$ are, up to equivalent renormings, L_2 and $L_{p,q}$. In the present note we consider the remaining values of p and q. Now the case $p = q$ (i.e., L_p) is treated in the Memoir of Johnson, Maurey, Schechtman and Tzafriri [11]; and since the non–separable, non–reflexive space $L_{p,\infty}$ contains a sublattice isomorphic to ℓ_∞ (hence L_∞), we will be concerned primarily with $p \neq q < \infty$.

In §2 we show that the main result of [5], stated above, also holds for $1 \leq q < 2 < p < \infty$. This is an unexpected extension of the results in [11], since $L_{p,q}$ is not 2–convex when $q < 2 < p$.

In §3 we give examples to show that in either of the cases $p < 2$ or $1 < p < q < \infty$ there are r.i. subspaces of $L_{p,q}$ that are not isomorphic to either L_2 or $L_{p,q}$. This is also surprising, as $L_{p,q}$ is 2–convex and q–concave when $2 < p < q < \infty$.

For the sake of brevity we will not repeat the arguments from [5] in their entirety, but rather simply indicate the necessary additions and alterations. The reader is referred to [5] and its references (especially [11] and [13]) for any unexplained terminology.

For $1 < p < \infty$ and $1 \leq q < \infty$ the Lorentz function space

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