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FUNCTIONS DEFINED BY CONTINUED FRACTIONS MEROMORPHIC CONTINUATION

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1. Introduction. A continued fraction

$$K\left(\frac{a_n}{b_n}\right) = \frac{a_1}{b_1} + \frac{a_2}{b_2} + \frac{a_3}{b_3} + \cdots\cdots = \cfrac{a_1}{b_1 + \cfrac{a_2}{b_2 + \cfrac{a_3}{b_3 + \cfrac{a_4}{b_4 + \ddots}}}}, \tag{1.1}$$

where a_n, $b_n \in \mathbf{C}$, $a_n \neq 0$ for all n, is an infinite process resembling a series in many ways. Corresponding to the partial sums of a series, we have the approximants of $K(a_n/b_n)$,

$$f_n = \mathop{K}_{m=1}^{n}\left(\frac{a_m}{b_m}\right) = \frac{a_1}{b_1} + \frac{a_2}{b_2} + \cdots + \frac{a_n}{b_n}, \qquad \text{for } n \geqq 0. \tag{1.2}$$

(Here $K_{m=1}^{0}(a_m/b_m) = 0$.) Further, we say that $K(a_n/b_n)$ converges to a value f, or that $K(a_n/b_n) = f$, if $\lim_{n\to\infty} f_n$ exists and is equal to f. (We permit convergence to ∞.)

Still in analogy with series, the elements a_n and b_n may be functions of a complex variable z. $K(a_n(z)/b_n(z))$ then defines a function of z in the subset $E \subseteq \mathbf{C}$ where $K(a_n(z)/b_n(z))$ converges. (Another way of defining functions by continued fractions, $K(a_n(z)/b_n(z))$, is by correspondence [3, §5.1]. In this paper, though, we shall use $f(z) = \lim_{n\to\infty} f_n(z)$ pointwise, for all z such that this limit exists.)

Finally, we also have modified approximants f_n^* of $K(a_n/b_n)$. They arise if we replace the n^{th} tail

$$\mathop{K}_{m=n+1}^{\infty}\left(\frac{a_m}{b_m}\right) = \frac{a_{n+1}}{b_{n+1}} + \frac{a_{n+2}}{b_{n+2}} + \cdots \tag{1.3}$$

of $K(a_n/b_n)$, not by 0 as in the ordinary approximants (1.2), but by a modifying factor w_n. That is, $f_0^* = w_0$ and

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