ROCKY MOUNTAIN
JOURNAL OF MATHEMATICS
Volume 15, Number 2, Spring 1985

GREEDY ALGORITHM AND COINAGE SYSTEMS

CELIA K. ROUSSEAU AND CECIL C. ROUSSEAU,

A. L. LIU, AND M. S. KLAMKIN

Given a coinage system with coins of denominations $1 = c_0 < c_1 < \cdots < c_k$, let $f_k(x)$ denote the minimum number of coins needed to give change for the amount x. By the principle of optimality

$$(1) \qquad f_k(x) = \min_{m \geq 0} \{m + f_{k-1}(x - mc_k)\},$$

where f_{k-1} refers to the corresponding function for the coinage system with denominations $c_0, \ldots, c_{k-1}$. When the greedy algorithm is applied to the minimization problem, the number of coins used to give change for the amount x is

$$(2) \qquad g_k(x) = [x/c_k] + g_{k-1}(x - [x/c_k]c_k).$$

Our paper gives a partial solution to the problem of characterizing those coinage systems for which $f_k = g_k$. Subsequent to the presentation of our paper, we found that a complete solution of the problem was obtained in [**1**].

Reference

1. M. J. Magazine, G. L. Nemhauser and L. E. Trotter, Jr., *When the greedy solution solves a class of knapsack problems*. Operations Research (Journal of the Operations Research Society of America) **23** (1975) 207–217.

C. K. and C. C. Rousseau: Memphis State University, Memphis
A. L. Liu and M. S. Klamkin: University of Alberta, Edmonton.

Received by the editors Julv 13, 1983