ROCKY MOUNTAIN
JOURNAL OF MATHEMATICS
Volume 14, Number 3, Summer 1984

SADDLE POINT APPROXIMATIONS IN n-TYPE EPIDEMICS AND CONTACT BIRTH PROCESSES

J. RADCLIFFE AND L. RASS

ABSTRACT. An n-type epidemic is considered. This model encompasses the measles, host-vector, and carrier-borne epidemics, and in addition rabies involving several species of animal. The saddle point approximation indicates that the asymptotic velocity of propagation in the deterministic model is the same as the minimum velocity, c_0, for which wave solutions exist in the deterministic model. It also suggests an approximation to the asymptotic expectation velocity in the stochastic model.

An n-type contact birth process is also defined. The analogue of the McKean connection between the distribution function of the position of furthest spread of the infection and the equations for the n-type simple epidemic is established. This suggests the asymptotic speed of translation of the distribution function is c_0.

1. Introduction. The equations for the spatial spread of a deterministic epidemic have been shown to only have wave solutions travelling with velocity greater than or equal to a critical velocity c_0. This has been proved by Atkinson and Reuter **[2]** for the case of constant infectivity with removals; and by Diekmann **[4]** for a more general model in which infectivity is allowed to vary with the time since infection, instead of infected individuals being removed at a steady rate. Use of the saddle point approximation, (Daniels **[3]**), indicates that the asymptotic velocity of propagation of infection in this model is in fact c_0. This has been proved rigourously by Aronson **[1]**, Diekmann **[5]** and Thieme **[11]**. The saddle point approximation gives the correct velocity of propagation using a comparatively simple method.

Let S_t be the position of the furthest spread of a one-type contact birth process. It is stated in Mollison **[7]** that $y(s, t) = P(S_t > s)$ satisfies the deterministic simple epidemic equations. It then follows that $y(s, t)$ propagates with speed c_0, where c_0 is the minimal possible speed at which

Received by the editors on September 29, 1983.

Key words and phrases: n-Type epidemic, host-vector and carrier-borne epidemics, rabies involving several species, contact birth process, velocity of propagation, saddle point approximation.