ROCKY MOUNTAIN
JOURNAL OF MATHEMATICS
Volume 14, Number 2, Spring 1984

ASYMPTOTIC BEHAVIOR OF SOLUTIONS OF AN nTH ORDER DIFFERENTIAL EQUATION

WILLIAM F. TRENCH

In this paper we give conditions which imply that the equation

$$u^{(n)} + f(t, u) = 0 \tag{1}$$

has a solution which behaves in a precisely specified way like a given polynomial of degree $< n$ as $t \to \infty$. We do not make the often imposed assumptions that f is continuous on $(0, \infty) \times (-\infty, \infty)$, or that it is majorized by a function which is nondecreasing in $|u|$. Moreover, our integral smallness conditions on f permit some of the improper integrals in question to converge conditionally.

Throughout the paper we write $f(t) = O(\psi(t))$ to indicate that $\overline{\lim}_{t\to\infty}|f(t)/\psi(t)| < \infty$, and $f(t) = o(\psi(t))$ to indicate that $\lim_{t\to\infty} f(t)/\psi(t) = 0$.

The following is our main theorem.

THEOREM 1. *Suppose k is an integer in $\{0, 1, \ldots, n-1\}$ and ϕ is positive, continuous, and nonincreasing on $[\bar{T}, \infty)$ for some $\bar{T} \geqq 0$; moreover, if $k \neq 0$, suppose there is a number γ such that*

$$\gamma < 1 \text{ and } t^{\gamma}\phi(t) \text{ is nondecreasing on } [\bar{T}, \infty). \tag{2}$$

Let p be a given polynomial of degree $<n$, and suppose there are constants $M > 0$ and $T_0 \geqq \bar{T}$ such that f is continuous on the set

$$\Omega = \{(t, u) \mid t \geqq T_0, |u - p(t)| \leqq M\phi(t)t^k\}, \tag{3}$$

and

$$|f(t, u_1) - f(t, u_2)| \leqq g(t)|u_1 - u_2| \text{ if } (t, u_i) \in \Omega,\ i = 1, 2, \tag{4}$$

where $g \in C[T_0, \infty)$,

$$\int^{\infty} s^{n-1} g(s)\, \phi(s)\, ds < \infty, \tag{5}$$

and

AMS(MOS) Classification: 34E99, 34C10.
Received by the editors on July 20, 1982, and in revised form on January 30, 1983.