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ON SOME PROPERTIES OF DOMAINS OF INTEGRAL OPERATORS

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SUMMARY. A construction of enlarging solid topological spaces of measurable functions is discussed. It is shown that both the domain and the extended domain of an integral operator are invariant under this construction.

1. Introduction. Let X be a measure space, $L^0 = L^0(X)$ be the vector space of measurable finite a.e. scalar-valued functions on X and let $A \subset L^0$ be a topological vector space. Denote $A^\# = \{u \in L^0; \{v \in A; |v(x)| \leqq |u(x)| \text{ a.e.}\} \text{ is bounded in } A\}$. If A is solid then $A \subset A^\#$, otherwise it may happen that $A^\# = \{0\}$. If A is a solid normed space then $A^\#$ is a space defined by a function norm in the sense of [1].

In this paper we study the "functor"$^\#$ as applied to the domain $\mathscr{D}_K$ and the extended domain $\tilde{\mathscr{D}}_K$ of an integral operator K. The conclusion is that both domains are preserved by $^\#$, (Theorem 4.1 and theorem 4.2) in particular if K is defined on A then it is also defined on $A^\#$ and if K extends by continuity to a solid topological vector space A then it also extends by continuity to $A^\#$.

As a preliminary to theorem 4.2 we prove theorem 2.1 which is a new characterization of the space $\tilde{\mathscr{D}}_K$.

Example (4.5) seems to show that Theorem 4.2 is nontrivial; we do not know a proof of (4.5) which would not involve in one way or another the idea of that theorem.

The reference [2] is the background of all the results outlined in Section 2.

2. Notation and prelimaries. We assume that X is σ-finite, by subsets of X we mean measurable subsets, the measure on X we denote by dx and the measure of a set $E \subset X$ we note by $|E|$.

By a metric ρ we shall mean a translation invariant metric and we shall write $\rho(u) = \rho(u, 0)$.

The space L^0 of all measurable, scalar valued, finite a.e. functions on X

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