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A NOTE ON HIGHER DERIVATIONS AND ORDINARY POINTS OF CURVES

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ABSTRACT. In this note, we prove the following theorem: Let k be an algebraically closed field of arbitrary characteristic. Let C denote a reduced curve in $\mathbf{A}_k^n$ and let p be a point of C. Let R denote the local ring of C at p and let $\bar{R}$ denote the integral closure of R in its total quotient ring. Let $M_1, \ldots, M_h$ be the branches of C at p. Then p is an ordinary point of C if and only if the following two conditions are satisfied: (a) $\mathrm{Der}_k^q(R) \subseteqq \mathrm{Der}_k^q(\bar{R})$ for all $q \geqq 1$; (b) For t a common uniformizing parameter of C in R, there exists an x in the maximal ideal of R such that $\partial x/\partial t$ is a unit in $\bar{R}$, and $(\partial x/\partial t)$ Mod $M_i \neq (\partial x/\partial t)$ Mod$_j$ for all $1 \leqq i < j \leqq h$.

Introduction. Throughout this paper, we shall let k denote an algebraically closed field of arbitrary characteristic. We shall let C denote a reduced curve in $\mathbf{A}_k^n$ (affine n-space over k). Let p denote a point of C. In this paper, we wish to characterize when p is an ordinary point of C in terms of the higher derivations on the local ring R of C at p. We shall first show that C is unramified at p precisely when every higher order k-derivation on R extends to the integral closure $\bar{R}$. To the best of my knowledge this result was first proven by T. Bloom in [2] for irreducible curves over the complex numbers $\mathbf{C}$. This result was later generalized to arbitrary fields of characteristic zero by J. Becker in [1]. In this paper, we present a purely algebraic argument which works in any characteristic.

In the last part of this paper, we present straightforward differential conditions which guarantee C has distinct tangents at p.

Preliminaries. In this section, we shall present the definitions and basic notation which will be used throughout the rest of this paper. We shall let C denote a reduced curve in $\mathbf{A}_k^n$. To be more specific; $C = \mathrm{Spec}\{k[X_1, \ldots, X_n]/\mathfrak{A}\}$ where $\mathfrak{A}$ is a radical ideal, unmixed of height

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